Synthesis and Experiments of Inherently Balanced Umbrella Canopy Designs

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Abstract—This paper shows how umbrella canopies and umbrella canopy-like mechanisms can be designed inherently balanced. Inherently balanced means that the center of mass of the moving parts remains stationary at a single point for any position of the mechanism simply because of the specific design of the mechanism where no counter-masses or springs are needed. The advantages of a balanced canopy are that it can be opened and closed easily with minimal effort even if it has large dimensions and that it is inherently safe since the canvas cannot collapse or fall down. Also a shaking ground or floor does not affect an inherently balanced canopy. This is due to its dynamic force balance, allowing the canopy to be perfectly suitable as a retractable roof on for instance a truck on a bumpy road. In this study, three inherently balanced umbrella canopy designs are synthesized from principal vector linkages and their balance conditions are derived. Also a prototype was built and tested to show the feasibility of the design in practice.

Keywords: static balance, inherent force balance, umbrella canopy, retractable roof, principal vector linkage

I. Introduction

Umbrella canopies are used in outdoor spaces for protection from sun and rain. This can be for recreational purposes but also for rescue and shelter services. Characteristic of umbrella canopies is that they have a single central mast which holds a deployable and retractable canvas [7][8]. Therefore umbrella canopies have the advantage to be compact for storage and transportation, while they also enable open and closed protected spaces [6][5].

A limitation of current umbrella canopies is the weight of the canvas mechanism that has to be moved up and down to open and close the canopy. Especially when canopies become large of size, the canvas mechanism tends to become heavy, requiring significant effort for opening and closing and for keeping it in a certain pose. At the same time, the safety is at risk. If, perhaps by mistake, the canopy collapses, the mass falls down possibly on the people underneath.

A solution for this is to design the canopy mechanism to be statically balanced. The motion of a statically balanced mechanism is not affected by gravity and it remains stationary in all possible poses. Common approaches to design a mechanism statically balanced is by applying counter-masses [3] or by applying elastic springs [2]. These approaches however lead to, respectively, relatively heavy and complex solutions.

In this paper, it is proposed to synthesize balanced umbrella canopy(-like) mechanisms from principal vector linkages. Principal vector linkages are inherently balanced mechanism architectures where the links are assembled such that their common center-of-mass (CoM) is in a stationary point on one of the links [1]. These linkages are based on solely the essential conditions for balance and do not need counter-masses or springs. This synthesis approach has led already to successful results such as the synthesis of a high-speed dynamically balanced parallel manipulator [9] and the synthesis of a 2-degree-of-freedom (DoF) inherently balanced grasper mechanism [4].

The aim of this paper is to synthesize three different balanced umbrella canopy mechanisms from principal vector linkages, to investigate the various design possibilities and to show its feasibility in practice. First the three synthesized balanced umbrella canopy mechanisms are presented and their balance conditions are derived. Subsequently the various ways of arranging the canopy arms are investigated. A prototype balanced umbrella canopy mechanism is shown at the end with an experimental evaluation of its balance performance.

II. Synthesis of Balanced Umbrella Canopy-type Mechanisms

In this section three different inherently balanced mechanisms are synthesized as balanced umbrella canopies. First the design of a single-DoF umbrella canopy is shown and its balance parameters are derived. Subsequently a second version of a single-DoF umbrella canopy is proposed for which the balance parameters are derived as well. A two-DoF balanced umbrella canopy design is derived and shown at last and its balance parameters are derived.

A. Single DoF retractable umbrella canopy

Figure 1 shows a 1-DoF balanced retractable umbrella canopy where the upper hub can move up and down along the line of the central axis of the mast. This is different from other umbrella-like mechanisms where the upper hub is mounted on the mast while the lower hub moves along mast. The canopy is composed of five arms of each two
The common CoM of all moving parts is designed such that the common CoM is in the joint with the mast for all positions of the mechanism.

The design in Fig. 1 was synthesized from the 2-DoF principal vector linkage in Fig. 2, which is also known as a force balanced pantograph linkage [1][4]. It consists of four elements with a mass $m_i$ of which the CoM is generally located and described with parameters $p_i$ and $q_i$ as illustrated. Joints $P_1$ and $P_2$ are known as the principal points and $a_1$ and $a_2$ are known as the principal dimensions. These parameters define the principal vector linkage such that the common CoM of all four masses is located in joint $S$ for all motion of the mechanism. This means that the mechanism is force balanced and statically balanced\(^1\) with respect to joint $S$. Then, if joint $S$ is made stationary, for instance as being the joint with the mast as shown in Fig. 1, the mechanism becomes balanced with respect to the mast and to the surrounding environment as well. The conditions for which $S$ is the common CoM are named the balance conditions which are derived and explained in [1] and write:

\[
\begin{align*}
m_1 p_1 &= m_2 a_1 + m_3 p_3 \\
m_1 q_1 &= m_3 q_3 \\
m_2 p_2 &= m_1 a_2 + m_4 p_4 \\
m_2 q_2 &= m_4 q_4
\end{align*}
\]

(1)

Each arm of the canopy in Fig. 1 is a half of the balanced pantograph in Fig. 2 considering the pantograph to be divided about the line $SA$. This means that the canopy consists of five halves of a balanced pantograph. Furthermore, the links in the canopy design are shown mass symmetric, i.e. the link CoMs are located along the line through their joints for which $q_1 = q_2 = q_3 = q_4 = 0$. This is not necessary, but it simplifies the design significantly. Also the canopy arms are all equal, meaning that both sides of the balanced pantograph in Fig. 2 are equal with respect to line $SA$ with $m_2 = m_1$, $p_2 = p_1$, $m_3 = m_4$, $p_3 = p_4$, and $a_2 = a_1$. Also this is unnecessary but simplifies the design significantly. With these choices the balance conditions (1) reduce to the single force balance condition:

\[
m_1 (p_1 - a_1) = m_4 p_4
\]

(2)

As a result, Fig. 3 shows the design of a single arm of the canopy with its mass parameters. Here $O$ is the joint with the mast and joint $A$ is shown with revolute pairs in a slider element. This slider illustrates the motion of joint $A$ in the canopy in Fig. 1 where, due to the combination of the multiple arms, joints $A$ and the moving hub have translational motion without the need of a slider element. In Fig. 3 it is also shown that joints $O$ and $A$ can have an

\(^1\)Force balance means that the dynamic (reaction) forces are balanced while static balance means that gravity does not affect the mechanism motion. Force balanced mechanisms are also statically balanced while statically balanced mechanisms can also be not force balanced, for instance if balancing springs are used.
offset from the axis of the mast, which allows a practical design of the moving hub with its joints and of the joints with the mast.

Also the mass of the moving hub can be included in the balance by dividing its value by the number of arms and locating this partial mass in each arm’s joint $A$ by which it can be included in parameters $m_1$ and $p_1$ of the long links. The exact location of the hub’s CoM is not of importance, it can be located anywhere with respect to the hub. When the CoM of the hub is aligned with the centerline of the mast and it is in the same plane as the joints $A$, then the common CoM of the canopy is exactly in the center in between joints $O$ as indicated in Fig. 3. If the hub’s CoM is located elsewhere then the common CoM of the canopy still is in a stationary point in the mast, but at a different location.

When it is chosen to not simplify the design but to have each or some of the arms of the canopy be different and to have the CoMs of the elements off the line through their joints, the flexibility of the design is increased. While the latter is relatively simple by allowing nonzero $q$’s in the balance conditions (1), resulting in two balance conditions for each canopy arm, the former option is more challenging since it requires multiple arms to be considered together with their specific radial orientation about the mast. This issue is investigated further in Section III.

B. Single DoF umbrella canopy with inner space in closed form

Since there are various principal vector linkages, also various umbrella canopy mechanisms can be synthesized similarly as the 1-DoF balanced umbrella canopy mechanism in the previous section. Figure 4 shows a 1-DoF balanced umbrella canopy design with 5 arms and solely revolute joints that was synthesized from the 3-DoF principal vector linkage in Fig. (3.7) in [1]. As compared to the canopy in Fig. 1 it has two additional links per arm and it has a large extending moving hub that consists of the upper links of each arm which are rigidly connected together. The hub has translational motion along the line of the central axis of the mast. When the canopy is closed an enclosed space is formed underneath the canopy because of the size of the moving hub. In practice this could be useful for storage purposes. Also here the common CoM of all moving elements is in the joint with the mast for all motion.

Figure 5 shows the design of the arm of the canopy with its mass parameters, illustrated similarly as the arm in Fig. 3. Here however, link $A_1P_2$ is part of the moving hub and can only translate along with the slider in $P_2$, as if it was rigidly connected with the slider. This also means that because of parallelogram $A_1P_3B_1P_4$ link $P_1B_1$ solely translates as well. Links $A_1P_1$, $B_1P_2$, and $OB_1$ have rotational motion.

The balance conditions of the arm can be derived from the general force balance conditions (3.22) in [1] of the 3-DoF principal vector linkage. Therefore $q_1 = q_3 = q_{11} = q_{12} = q_{13} = q_{31} = q_{32} = q_{33} = 0$ to obtain mass symmetric links and $m_3 = m_1$, $p_3 = p_1$, $m_{31} = m_{11}$, $p_{31} = p_{11}$, $m_{13} = m_{33}$, $p_{13} = p_{33}$, $m_{32} = m_{12}$, $p_{32} = p_{12}$, and $a_{23} = a_{21}$ to obtain equal arms on each side. With these parameters the force balance condition of the canopy arm in Fig. 5 becomes

$$m_1(p_1 - a_1) = (m_2 + m_{11} + m_{12})a_1 + m_{12}p_{12} + m_{33}p_{33}$$  \(3\)

It is interesting to note that the locations of $m_2$ and $m_{11}$ in their links $A_1P_2$ and $P_1B_1$, respectively, are not defined. Indeed, they can be located anywhere within their link for
balance since both the links have solely translational motion. The exact location of the common CoM depends on the locations of \( m_2 \) and \( m_{11} \), but for any choice it is in a stationary point for all positions of the canopy mechanism. The condition for which the common CoM is located precisely in between the joints \( O \) with the mast is also derived from the general force balance conditions and writes

\[
m_2 p_2^a = m_1 a_{21} + m_{11} p_{11} \tag{4}
\]

Here \( p_2^a \) is the distance from \( P_2 \) to \( m_2 \) in the illustrated direction (see: [1] for further explanation). If \( m_2 \) would be on this line, then the common CoM would be exactly in between the joints \( O \) as illustrated. Since \( m_2 \) is below this line, also the common CoM will be below the indicated point.

C. Two-DoF balanced deployable umbrella canopy

As a third design of a balanced umbrella canopy mechanism Fig. 6 shows a version which has 2-DoF motion. Also this design has only revolute joints and is shown with 5 arms which are not only connected together with a moving hub on top and in the joints with the mast, but also with a moving hub in the middle. Both moving hubs have translational motion along the line of the central axis of the mast. The distance between the middle hub and the joint with the mast determines the orientation of the most extended links. For the pose in Fig. 6b the distance between the middle hub and the joint with the mast is maximal which results in the most extended links being vertical. The distance between the top hub and the middle hub determines the retraction of the canopy mechanism. Figure 6c shows that when this distance becomes maximal the canopy is fully retracted. It can be said that this canopy mechanism combines the features of both of the canopies in Fig. 1 and Fig. 4, being fully retractable and of having an inner space in closed form.

The design of the balanced umbrella canopy mechanism in Fig. 6 was synthesized from a 4-DoF principal vector linkage in Fig. (3.23) in [1] in a similar way as the previous two canopy designs. Fig. 7 illustrates the design of the arm of the canopy with its mass parameters. The balance conditions of the arm can be derived from the general force balance conditions (3.67) in [1]. Therefore

\[
q_1 = q_4 = q_{11} = q_{12} = q_{13} = q_{14} = q_{21} = q_{22} = q_{31} = q_{32} = q_{41} = q_{42} = q_{43} = q_{44} = 0
\]

to obtain mass symmetric links and \( m_4 = m_1, p_4 = p_1, m_3 = m_2, p_2 = p_2^a - p_{21} = p_3 = p_3^a - p_{31}, m_{14} = m_{44}, p_{14} = p_{44}, m_{21} = m_{31}, p_{21} = p_{31}, m_{22} = m_{32}, p_{22} = p_{32}, m_{41} = m_{11}, p_{41} = p_{11}, m_{42} = m_{12}, p_{42} = p_{12}, m_{43} = m_{13}, p_{43} = p_{13}, \) to obtain equal arms on each side. With these parameters the two force balance conditions of the canopy arm in Fig. 7 become

\[
m_1 p_1 = (m_1 + 2m_2 + m_{11} + m_{12} + m_{13} + 2m_{31} + m_{32}) a_1 + m_{12} p_{12} + m_{13} p_{13} + m_{44} p_{44}
\]
\[ m_2 p_2 = m_1 a_{21} + m_{11} p_{11} - m_{31} p_{31} - m_{32} p_{32} - (5) \]

\[ (m_1 + m_2 + m_{11} + m_{12}) a_{23} \]

Also here the mass of each moving hub can be included by dividing the mass values by the number of arms and including these partial masses in the joints of the hubs by which they can be included in the mass parameters of the connecting links.

III. Arm Configurations

![Diagram of arm configurations](image)

**Fig. 8.** Top view of various radial locations of multiple umbrella canopy arms about the mast for balance.

The balanced umbrella canopy designs in Fig. 1, Fig. 4, and Fig. 6 can have any number of arms, not limited to 5 as illustrated. Figure 8 shows a brief investigation of various conditions for multiple arms and their radial locations about the mast. Figure 8a shows that if the mast of the canopy is perfectly vertical (i.e. the mast is aligned with the direction of gravity) any number of arbitrarily designed balanced arms can be arbitrarily located around the mast to obtain a statically balanced canopy. If the mast is not perfectly vertical then the canopy arms always need to be located about the mast in some symmetric way. With two arms, the only possible solution is shown in Fig. 8b where the arms act oppositely of one another. For such a configuration it is possible to apply the balanced pantograph in Fig. 2 and the general principal vector linkages from which the canopies in Fig. 4 and Fig. 6 were derived without any adaptations. When three arms are used, they could be equally distributed about the mast as shown in Fig. 8c. Then arms 2 and 3 balance another about the y-axis while they together balance arm 1 about the x-axis. The same is true when arms 2 and 3 are located symmetrically about the y-axis as shown in Fig. 8d. Here arms 2 and 3 have to be equal for balance about the y-axis but they have to be lighter than arm 1 to obtain balance about the x-axis.

Figure 8e shows a configuration of a canopy mechanism with 4 arms in pairs of two. Since each pair is individually balanced as in the configuration in Fig. 8b, the relative location of the two pairs is arbitrary. One specific solution is to locate the pairs orthogonally, indicated with the dashed lines for which the four arms are equally distributed about the mast.

In Fig. 8f a configuration with 4 arms symmetrically arranged about line \( t_1 \) is shown where arms 1 and 2 are equal and arms 3 and 4 are equal. Here arms 3 and 4 have to be a little lighter than arms 1 and 2 to have the pairs balance another. This configuration can also be interpreted as that the 4 arms are symmetrically arranged about line \( t_2 \) with arms 1 and 4 being equal and arms 2 and 3 being equal.

With the increasing number of arms, also the number of possible arm configurations increase. Figure 8g shows a configuration of 5 arms equally distributed about the mast, equal to the canopy designs in Fig. 1, Fig. 4, and Fig. 6. \( t_1 \), \( t_2 \), and \( t_3 \) are the symmetry axes about which the arms can be analyzed for balance. The easiest way however is to project the design of each arm onto the orthogonal xy-planes as for Fig. 8c where balance is obtained within each of the planes. With such an approach it is also possible to find configurations as in Fig. 8h where the arms may appear arbitrarily located about the mast, but in fact act about the line \( t_1 \) for balance. The configurations illustrated in Fig. 8 are just a few of the wide variety of possibilities.

IV. Experimental evaluation

The balanced umbrella canopy mechanism in Fig. 1 was developed as a prototype which is shown in three poses in Fig. 9. Instead of 5 arms the prototype has 4 arms that are equally distributed about the mast. The moving hub of POM material with 4 joints is best visible in Fig. 9a. The moving links were made of MDF wood with a cross section of 8.2 mm wide and 20.5 mm high and the mast was made of aluminium 5-40 mm tube, 700 mm long. The hub of the joint with the mast is also of POM and the M6 bolts, nuts, and washers are of steel.

First the short moving links were manufactured, which
were calculated from Fig. 10a as $m_{i} = p_{i} - a_{i}$ and $m_{hub} = 43.52$ g and $p_{4} = (30.40a_{1}/2+13.12a_{1})/m_{4} = 65.08$ mm. Then from the force balance condition (2) the condition of the parameters of element $AP_{1}$ were calculated as $m_{1}(p_{1} - a_{1}) = m_{4}p_{4} = 43.52 \cdot 65.08 = 2832.38$ g-mm.

The moving hub was designed, produced, and weighted together with 4 M6 bolts, 4 nuts, and 12 washers of which the total mass was measured as 149.70 g. Considering the missing mass in the $\varnothing 6$ mm holes in the long link at the four joints $A$ with the hub, the hub mass is derived as $m_{hub} = 149.70 + 4(-0.176) = 149.00$ g. Subsequently the parameters of link 1 were calculated according the model in Fig. 10b. Here 1/4 of the moving hub mass is modeled in joint $A$ and the mass $m_{link1}$ of the link element is located halfway the length of the link at $l_{1}/2$, which is at a distance $s + p_{1}$ from $P_{1}$ as illustrated. This length is also calculated as $l_{1} = 2(a_{1} + p_{1} + s + d)$ with $d = 9$ mm the link length extending from $A$. The link has a mass distribution of $p = 0.1278$ g/mm with which $m_{link1} = p l_{1}$. Then with

$$\frac{m_{hub}}{4}(a_{1} + p_{1}) = m_{link1}s$$

and substituting $m_{link1}$ and Eq. 2 for $p_{1}$, a quadratic equation for $l_{1}$ is obtained as

$$\frac{p}{2} l_{1}^{2} - (2a_{1} + d)pl_{1} - m_{4}p_{4} - a_{1}m_{hub} = 0$$

From this equation $l_{1} = 661.32$ mm is found. Subsequently the other design parameters of the link are found as $m_{1} = 121.77$ g and $p_{1} = 123.26$ mm with $m_{link1} = 84.52$ g, and $s = 98.40$ mm.

Link 1 was manufactured with $l_{1} = 668$ mm with $m_{link1} = 85.42$ g, which is slightly larger than the computed value, in order to be able to make adjustments after the assembly. The length $a_{1}$ of this link, however, turned
out to be 99.3 mm, which is 0.7 mm shorter than the required value of 100.0 mm. During assembly, the bolt-nut connections were kept as loose as possible to keep friction low. After the assembly of the mechanism, a transparent plastic foil was spanned about the canopy as a canvas of which the mass was neglected.

As a first experiment the balance was verified by moving the prototype by hand, which was experienced as being very easy even with a soft touch. The prototype remained stationary in each position as is expected for balance. A video showing the balanced motion of the prototype can be found at http://mecart.iyte.edu.tr/projects.html.

To investigate the balance performance in more detail, several measurements were carried out of the prototype in an experimental setup. The transparent foil was removed. For different poses of the prototype the forces were measured that are needed to start moving the hub up and to start moving the hub down. These forces are expected to be equal for balance while their values represent the amount of friction.

The total span height of the prototype between the joint and the moving hub is 20 cm. Measurements were taken at the distances between the moving hub and the mast joint of 3 cm, 6.5 cm, 10 cm, 13.5 cm and 17 cm, respectively. The precision of the hand scale was 0.01 kg, which showed to be of limited accuracy for this application. It also showed rather challenging to detect the exact start of the motion of the hub, which affected the repeatability negatively. Table I shows the experimental results of the forces that were measured for each experiment carried out three times. Due to the limited precision of the hand scale the force down at 3 cm could not be measured.

Analysis of the data in Table I shows that the canopy can be moved with relatively small forces. However it also shows that for all positions a higher force is necessary to move the hub upwards than to have the hub move downwards. This implies that the prototype may not be perfectly balanced, which is indeed true. The main cause is the 0.7 mm (0.7%) difference of the length a1 in the long link. Although this may seem a small error, since linkage APSP1 in Fig. 2 is not anymore an exact parallelogram, its balance is affected directly. Since length a1 is larger in the short links, the short links are always a little more vertical than the long links. Therefore the mass of the long links is always a little bit too high and it has to be slightly pushed up for moving the hub upwards, for which more force is required. This effect is reinforced with the long links being manufactured 6.68 mm longer than calculated, for which their mass nhink showed to be about 0.9 g (over 1%) heavier. Since the error in the link length has more influence on the balance when the links are more vertical, higher forces are required for the moving the canopy at higher points. This is also observed from the measurement data in Table I, however the reduced force transmission within the linkage when the links become nearly vertical may have been a more significant cause for this.

V. Discussion

The balanced umbrella canopy mechanisms in Fig. 1, Fig. 4, and Fig. 6 are not only statically balanced, but also they are also shaking force and shaking moment balanced. While this is not of importance for the relatively slow opening and closing motion of the canopy for which the dynamic reaction forces and moments on the mast are negligible, in case of a shaking base or ground, for instance during transportation (if not closed) along a bumpy road or in case of an earthquake, this property increases the inherent safety of the canopy.

In Fig. 8 variations on the canopy arm locations about the mast were shown. Also each canopy arm has a wide variety of possible designs. To mention just two of them, Fig. 11a shows that a balanced canopy arm can be shifted individually, not only away from the mast, but also along the direction of the mast and Fig. 11b shows that a balanced canopy arm can be scaled individually with its mass being inversely scaled as well. In [1] (Section (3.2.4)) other applicable kinematic variations can be found. For instance parallel links do not necessarily need to be pivoted in $P_1$.  

<table>
<thead>
<tr>
<th>Height (cm)</th>
<th>Force down (kg)</th>
<th>Force up (kg)</th>
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<tbody>
<tr>
<td>3</td>
<td>#1 0.22</td>
<td>#2 0.16</td>
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</tr>
<tr>
<td>17</td>
<td>#1 0.34</td>
<td>#2 0.32</td>
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TABLE I. Experimental results of the prototype canopy by measurements of the forces needed to start moving the hub up and down from a certain pose.

Fig. 11. Two possible kinematic variations of balanced canopy arm configurations: a) shifted arm; b) scaled arm.
but under certain conditions they can be placed elsewhere, maintaining balance.

Although the balanced mechanisms in Fig. 1, Fig. 4, and Fig. 6 were synthesized for the purpose of umbrella canopies, they could as well be applied in many other designs of balanced devices and machinery.

VI. Conclusions

In this paper inherently balanced umbrella canopy mechanisms were synthesized and investigated. A balanced umbrella canopy mechanism with single DoF was synthesized from a 2-DoF principal vector linkage, able to retract fully. From a 3-DoF principal vector linkage another 1-DoF balanced umbrella canopy mechanism was synthesized which in closed state leaves an enclosed inner space. From a 4-DoF principal vector linkage a 2-DoF balanced umbrella canopy mechanism was synthesized which both can be fully retracted and can be closed leaving an enclosed inner space.

Some of the wide variety of design possibilities were explored by investigating the number of arms and their radial distributions about the central shaft and some variations of their relative positions along the mast. These possibilities allow the designs to be adapted to the need in various applications and circumstances. For all canopy mechanisms the design parameters for balance were derived and a prototype model of a 1-DoF balanced umbrella canopy was built and experimentally tested to prove and show its feasibility in practice.

References