Kinematics and Dynamics of Motion of the Docking Manipulator of Spacecraft

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Abstract

The work is composed of the basic dynamic equations of the new spatial docking manipulator equations of the new docking manipulator 6-RUPU consist of a platform and six legs. In each leg there are two Hooke joints, rotational and translational pairs. Dynamics of docking manipulator of spacecraft are described by the method of Lagrange- Euler six nonlinear differential equations of the second order.

Keywords: Theory of orbital docking, spacecraft, kinematics of manipulators, dynamics of motion.

1.Introduction

The extrapolation of space has led to the development of new directions in the science of machines and mechanism. The development of space vehicles and stations required the development of the problem of computing the kinematics and dynamics to solve the spaceship. Docking device is intended for direct connection of spacecraft in orbit.

1.1 Preface

Up to date standart spaceflight is increasingly used orbital docking. Theory and practice orbital dock turned into a scientific field of space station with two docking assemble providing a rigid connection with the formation of a sealed tunnel that leads to effective operation of the space complex. The first docking devices were created for spacecraft (SC) “Union” (USSR) with automatic approach and docking, as well as with the participation of astronauts and the “Gemini” (USA) with mandatory participation of the crew in the navigate.

The implementation of the first projects of the “Union- Union” and the lunar program “Apollo” has led to the development of technology dock. Docking was carried out using the active docking mechanism with pin and passive with the receiving cone.

For the implementation of international project have used the principle of androgyny peripheral docking aggregates – principle of reversal symmetry of the location of interacting and connecting elements.The successful implementation of the project ''Union – Apollo '' was crowned by two-time docking in space, and each of androgenic units performe alternately active and passive role.

Docking units are concentrated a large number of interacting mechanisms. The complexity of operations and versatility of the end effectors complicates the solution of the problems of analysis, synthesis, kinematics and dynamics of spatial mechanisms docking units. The action of external forces, the location and dimensions of the guide elements define the interaction after the first touch. Their choice is determined by the initial conditions of the docking provided by the systems to approach the SC to the end site. Classification of docking devices, analysis and synthesis of the basic mechanisms and individual parameters of the structures allows to generate a schematic diagram of the docking units. Great attention is paid to the design locks the docking frame. It is known that most of the mechanisms connecting devices work in open space, so you need to consider additional terms in the design of the main mechanisms , components and elements.You must also take into account the reliability of the docking device.

The subsequent design stage is dock dynamics.Space dock has set the task dynamics shock absorption at arbitrary colision of two free-space designs.Dynamic analysis uses elements of the classical theory of impact of rigid bodies and equations, taking into accout finite deformation of shock absorbers and designs.

The creation of a docking device required the development of new mechanisms and elements.On the basis of electromechanical dampers established electromechanical docking device. By cushioning the rotor brake may be dispersed in units of milliseconds up to a speed of several thousand revolutions per minute.

1.2 Functions of the Docking Devices of Spacecraft

Navigated spacecraft (SC) are brought to touch a certain speed and position, followed by a docking process that culminates in a rigid connection.After the flight uncoupling occurs by the release of mechanical linkages and repulsion SC. Connection SC is realized by two docking assembles (DA) called docking device (DD).

If all operations for docking and undocking is performed by a single unit, such a unit is called active and the other passive respectively.The unit, which can be active or passive is called androgenic.

Docking of spacecraft (SC) can be produced also by means of the intermediate platform type of the manipulator, which provides mechanical grip SC.

The main objective DD is SC mechanical connection with the possibility of multiple docking and undocking.DD performs three basic functions : docking, maintaining the docking state, undocking. DD must meet two basic requirements high reliability and low weight.

Note basic operations for DD:

1. depreciation;
2. payment of penalty;
3. the coupling;
4. alignment;
5. contraction;
6. the final alignment;
7. rigid connection;
8. disconnection;
9. the repulsion SC;
10. the control SC.

DD also performs additional operations:

1. open covers;
2. tunnel release;
3. leak test;
4. pressure relief from the tunnel.

Docking and undocking performed:

1. automatically;
2. with remote control from astronaut's control panel;
3. with remote control from the Ground;
4. manually.

DD consists of two units.Mechanism DA includes:

1. housing;
2. docking mechanism (DM);
3. docking frame.

Docking frames of the two units are interconnected by means of locks.The frame elements provide the connection, sealing and dedocking.

1.3 Initial Conditions of the Dock

When docking SC converging so as to be positioned coaxially position docking units and zero speed for the rest of the linear and angular coordinates.The possible values of the relative coordinates defining the deviation from the aligned position, and their first derivatives in mechanical contact are called initial conditions of the dock.The relative position and velocity of the translational movement and orientation of both units usually are maintained using traffic control system active SC.

Under automatic control uses a radar system that measures the relative linear and angular coordinates and their derivatives.When the pilot control used optical viewfinders and cameras active on SC, the target and the optical indices of the spatial type to passive.These targets allow you to measure both linear and angular deviations from SC coaxially position.

Alignment sight and targets are consistent with coaxially located DA.The pilot of the active device while watching the target on the screen of the viewfinder, the effect on the Executive bodies of the ship by using three degrees of freedom control handles so that they retain a coaxial position with passive set rate.

Passive SC during the approach supports the angular stabilization, and in the presence of automatic radiolocator-orientation to active. Active SC gradually reduces the range of initial deflections and velocities.

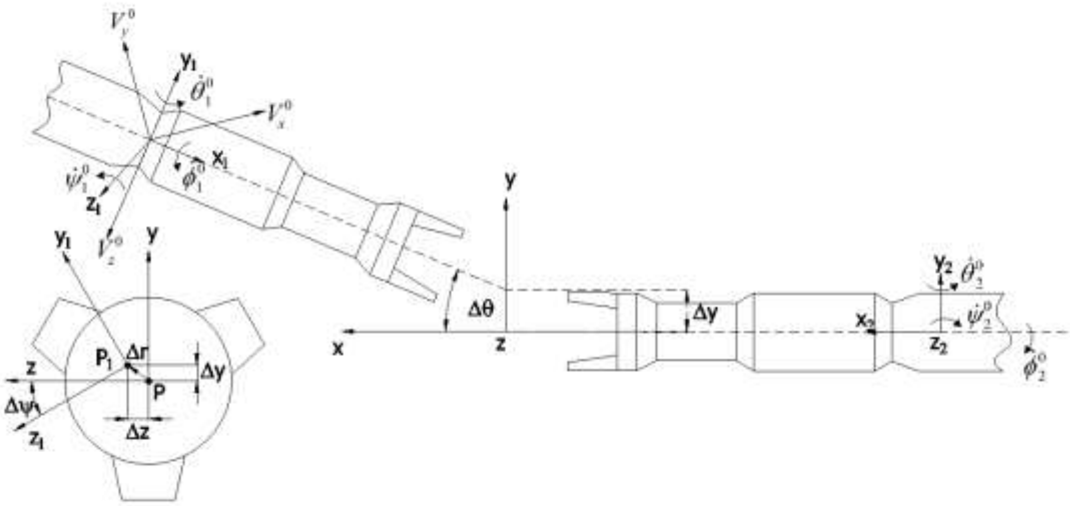

Figure 1: x, y, z - Coordinate System of Reference Parameter’s Initial Conditions

Deviations from a coaxial position convenient to set (Fig. 1) two linear coordiantes Δy and Δz on y and z axes associated with passive DA and two flat angles $\Delta\psi$ and $\Delta\theta$, also V_x, V_y, V_z – relative velocity; $\Delta r(\Delta y, \Delta z)$ is the lateral displacements; $\Delta\theta(\Delta\psi)$- is the angle between the x axis and the projection of the axis x_1 on the xy(xz) plane.

Angle $\Delta\phi$ of intersection of the planes x_1y_1 and x_2y_2 in the plane yz.The relative velocities of the centers of mass V_x^0 , V_y^0 , V_z^0 are defined in the projections on the axes x, y, z and angular velocity $\dot{\phi}_1^0$, $\dot{\psi}_1^0$,$\dot{\theta}_1^0$ and $\dot{\phi}_2^0$, $\dot{\psi}_2^0$,$\dot{\theta}_2^0$ in projections on the associated axis.

The total deviation of the docking units coaxially from the provisions consists of errors:

1. install DA;
2. measurement;
3. dynamic arising in the control process.

2.The Equations of Motion of Spatial Manipulator

The problem of dynamic control of manipulators is to determine the dynamic response using the inpulse reaction forces and inertia forces acting on the links of the mechanism.The problem of control is to obtain the basic equations of the dynamics of the manipulator in the form of a dynamic model with subsequent determination of the control laws to obtain the necessary response.The paper considers the issues of obtaining a physical model of the dynamics of a spatial parallel manipulator structures of the docking device spacecraft.

Spatial parallel structure manipulator is presented in the form of six open-loop kinematic and dynamic circuits. Each leg circuit is connected to the rotational, translational and two universal Hooke's joints (Fig. 2).

In statics and dynamics for the considered manipulator are determined values of bringing moments that allow you to achieve the required force and moment of docking device spacecraft.Thus, we deal with the inverse problem of dynamics of a spatial parallel structure manipulator, i.e. the problem of computing bringing moments needed to obtain the desired generalized coordinates, velocities by six nonlinear differantial equations of the second order.Differantial equations defined by the following moments and forces: forces and moments of reaction inertial forces and moments; centrifugal forces and moments; load forces and moments.

2.1 The Equation of the Dynamics of Manipulators Parallel Structure

Consider applying the method of Lagrange-Euler for spatial manipulators with parallel structure in combination with screw by recurrent expressions in open-loop kinematics [1,2].If you do not take account of the elastic force and gravity links each leg , as well as friction in the kinematic pairs of docking manipulator [1,2], the value of the total potential energy can be taken to zero[3, 4].Then the equation of the dynamics of a spatial parallel manipulator structure with six degrees of freedom can be written as:

$$\frac{\partial}{\partial t}\left[\frac{\partial T}{\partial \omega_i}\right]-\frac{\partial T}{\partial \alpha_i}=Q_i \tag{2.1}$$

Where: Q_i – the generalized forces and moments; T – the kinetic energy of the end effector and the input links of the mechanical system; w_i – the values of the angular velocities of the input links of the manipulator; α_i – the values of the generalized coordinates of the manipulator $i = 1,\ldots,6$ – is the number of generalized coordinates of the manipulator.

2.2 Definition of Generalized Moments of the Mechanical System

In the time of drawing up mathematical models of the dynamics of manipulators make use of the theory of screw calculus.Forces and moments of resistance forces applied to the ring of the docking device, are presented in the form of a power screw, towards the center of the ring:

$$E=\bar{F}+\omega\bar{M} \tag{2.2}$$

where: $\bar{F}$ – vector of forces of resistance; $\bar{M}$ – the moment vector $\bar{F}$ relative to the fixed coordinate system; ω– the operator of Clifford ($\omega^2 = 0$).

Screw calculus is the vector algebra of dual vectors.Each power screw (2.2) can be described in space by using three dual coordinates $E(\widetilde{M_x},\widetilde{M_y},\widetilde{M_z})$. Then each of the dual coordinate of the power screw (2.2) is devided into two physical parts:

$$M_x=F_x+\omega M_x; \quad M_y=F_y+\omega M_y; \quad M_z=F_z+\omega M_z \tag{2.3}$$

Six dual physical forces and moments of forces of the system (2.3) are the components of the power screw $E(F_x, F_y, F_z, M_x, M_y, M_z)$, where $\bar{F}(F_x, F_y, F_z)$ – are components of the vector of force and $\bar{M}(M_x, M_y, M_z)$ –are moments from the components of forces relatively coordinate axes x,y,z. We write the equation of the power screw directed through the center of the ring of docking device in the form:

$$\bar{M} = \overline{\rho_c} \text{ x } \bar{F} \tag{2.4}$$

where: $\overline{\rho_c(x_c, y_c, z_c)}$ – the radius of vector of the point of application of force F; x_c, y_c, z_c – Cartesian coordinates of the radius vector of the center of the ring of the docking device.

On the basis of (2.4) we write the equation of the power screw in space, directed through the center of the ring of the docking device:

$$M_x = F_z y_c - F_y z_c; \quad M_y = F_x z_c - F_z x_c; \quad M_z = F_y x_c - F_x y_c \tag{2.5}$$

Generalized torques of input links are determined from the condition of equality of the elementary works of these moments, when infinitesimal changes of generalized coordinate $\alpha_i$and a fixed value of time t, the work of the external forces and moments the resistance forces applied to the ring of the docking device, the possible displacements of their points of appication:

$$\sum_{i=1}^{6} Q_i \delta\alpha_i = \begin{Bmatrix} F_x \delta x_c + F_y \delta y_c + F_z \delta z_c \\ +M_x \delta\psi + M_y \delta\theta + M_z \delta\phi \end{Bmatrix} \tag{2.6}$$

Where: δx_c, δy_c, δz_c –the projection of possible displacements of the center of the ring for a docking device on the coordinate axis;

$\delta\psi$, $\delta\theta$, $\delta\phi$ – the possible values of the angular rotations of the ring of the manipulator around the coordinate axes.

During the control of multimobility manipulators, typically identify alternately elementary work of all external forces when you change one of the generalized coordinates α_i , fixed values of the other generalized coordinates:

$$\begin{gathered} \delta x_c = \frac{\partial x_c}{\partial \alpha_i} \delta\alpha_i; \quad \delta y_c = \frac{\partial y_c}{\partial \alpha_i} \delta\alpha_i; \quad \delta z_c = \frac{\partial z_c}{\partial \alpha_i} \delta\alpha_i \\ \delta\psi = \frac{\partial \psi}{\partial \alpha_i} \delta\alpha_i; \quad \delta\theta = \frac{\partial \theta}{\partial \alpha_i} \delta\alpha_i; \quad \delta\phi = \frac{\partial \phi}{\partial \alpha_i} \delta\alpha_i \end{gathered} \tag{2.7}$$

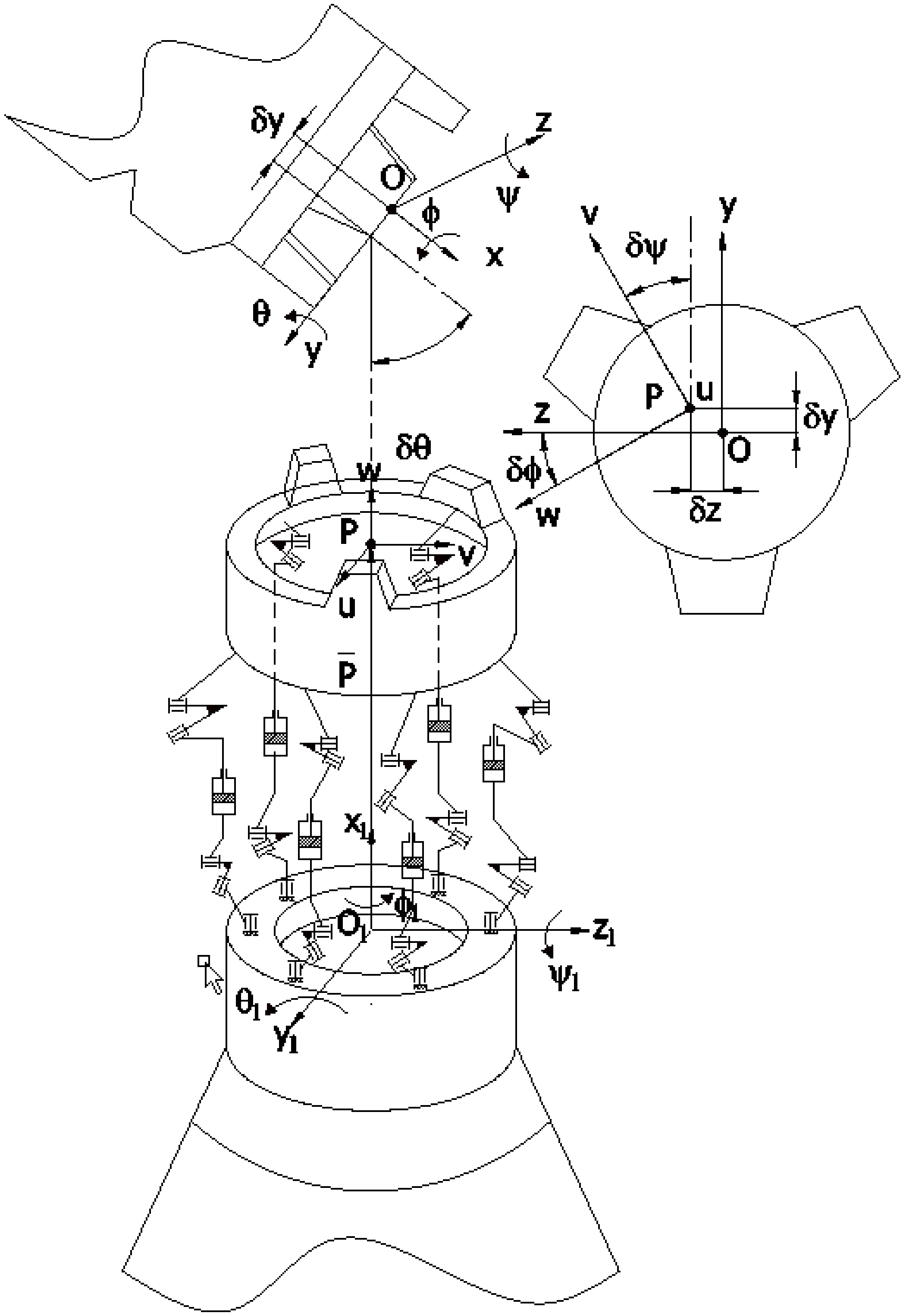

Figure 2: Docking Manipulator of Spacecraft

Therefore, fixing the values of five generalized coordinates, we obtain a mechanism with one degree of freedom, so far each combination of fixed generalized coordinate values of the generalized moment forces will be equal to the value bringing moment forces.Using (2.6) and (2.7), we obtain :

$$M_{bi}=\left\{\begin{array}{l} F_x\frac{\partial x_c}{\partial\alpha_i}+F_y\frac{\partial y_c}{\partial\alpha_i}+F_z\frac{\partial z_c}{\partial\alpha_i} \\ +M_x\frac{\partial\psi}{\partial\alpha_i}+M_y\frac{\partial\theta}{\partial\alpha_i}+M_z\frac{\partial\phi}{\partial\alpha_i}\end{array}\right\};i=1,\ .\ .\ .\ ,6 \qquad (2.8)$$

Note that the moment the Lagrangian is equal to the sum of possible works forces and moments the forces applied to the ring of the docking device, when you change only one generalized coordinate.

3.The Equations of Motion of Spatial Docking 6-RUPU Manipulator of Spacecraft

Consider equations of motion of a spatial parallel manipulator structure with variable inertia coefficients that depend on the position of spacecraft types 6-RUPU (Fig.2). Write the equation of motion for the Lagrange-Euler (2.1) as follows:

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial\omega_i}\right)-\frac{\partial T}{\partial\alpha_i}=M_{bi}\quad ;\ i=1,\ .\ .\ .\ ,6. \qquad (3.1)$$

The kinematic energy of the system is defined under the assumption that the input links of the legs are balanced and the masses and moments of inertia of the rod can be neglected, because the ring is hollow and six rods are arranged symmetrically relative to the ring.According to the adopted assumptions, we write the kinetic energy of the mechanical system taking into account the output ring of the manipulator:

$$T=0.5\left[\begin{array}{l}\sum_{i=1}^{6}J_i\omega_i^2+m\left(\dot{x}_c{}^2+\dot{y}_c{}^2+\dot{z}_c{}^2\right) \\ +J_x\omega_x^2+J_y\omega_y^2+J_z\omega_z^2\end{array}\right] \qquad (3.2)$$

where: J_i − respectively the moments of inertia of the input links relative to the axis of rotation of the rotational pairs;

w_i − angular velocities of the input links;

m − the mass of ring of the docking device;

$\dot{x}_c, \dot{y}_c, \dot{z}_c$ − the projections of the velocity of the center of gravity of the ring;

J_x, J_y, J_z − the moments of inertia of the ring relative to the principal axes of inertia;

w_x, w_y, w_z − the projection of the angular velocity on these main axes of inertia.

Note that, the coordinate axes are chosen so, that all the centrifugal moments of inertia are equal to zero. Translational motion of the center and orientation of the docking ring of the spatial manipulator are functions of the six angles of rotation of the input links.

$$\begin{gathered} x_c = x_c(\alpha_i); \quad y_c = y_c(\alpha_i); \quad z_c = z_c(\alpha_i) \\ \psi = \psi(\alpha_i); \quad \theta = \theta(\alpha_i); \quad \phi = \phi(\alpha_i) \end{gathered} \tag{3.3}$$

Differentiation of expressions (3.3) in time gives the projection of the velocity of the center of the ring and the projection of the instantaneous angular velocity on these main coordinate axes:

$$\begin{gathered} \dot{x}_c = \sum_{i=1}^{6} \frac{\partial x_c}{\partial \alpha_i} \omega_i; \quad \dot{y}_c = \sum_{i=1}^{6} \frac{\partial y_c}{\partial \alpha_i} \omega_i; \quad \dot{z}_c = \sum_{i=1}^{6} \frac{\partial z_c}{\partial \alpha_i} \omega_i \\ \omega_x = \sum_{i=1}^{6} \frac{\partial \psi}{\partial \alpha_i} \omega_i; \quad \omega_y = \sum_{i=1}^{6} \frac{\partial \theta}{\partial \alpha_i} \omega_i; \quad \omega_z = \sum_{i=1}^{6} \frac{\partial \phi}{\partial \alpha_i} \omega_i \end{gathered} \tag{3.4}$$

where the partial derivative (3.4) correspond to the ratio of output parameters of the ring to the input links. We introduce the notation:

$$\begin{gathered} U_{xi} = \frac{\partial x_c}{\partial \alpha_i}; \quad U_{yi} = \frac{\partial y_c}{\partial \alpha_i}; \quad U_{zi} = \frac{\partial z_c}{\partial \alpha_i} \\ U_{\psi i} = \frac{\partial \psi}{\partial \alpha_i}; \quad U_{\theta i} = \frac{\partial \theta}{\partial \alpha_i}; \quad U_{\phi i} = \frac{\partial \phi}{\partial \alpha_i} \end{gathered} \tag{3.5}$$

with each i for a private derivative correspond to five fixed independent of the other input parameters.

Taking into account (3.5) the expression (3.4) has the form:

$$\dot{x}_c = \sum_{i=1}^{6} U_{xi}\omega_i; \quad \dot{y}_c = \sum_{i=1}^{6} U_{yi}\omega_i; \quad \dot{z}_c = \sum_{i=1}^{6} U_{zi}\omega_i$$
$$\omega_x = \sum_{i=1}^{6} U_{\psi i}\omega_i; \quad \omega_y = \sum_{i=1}^{6} U_{\theta i}\omega_i; \quad \omega_z = \sum_{i=1}^{6} U_{\phi i}\omega_i \qquad (3.6)$$

Thus, the problem of determining the relations in equation (3.6) is reduced to the problem of kinematic analysis of mechanisms with one degree of freedom.

After substitution of the values of output parameters (3.6) in equation (3.2) and after elementary transformations, we get the expression of the kinematic energy of six degree of freedom of the spatial docking manipulator with parallel structure:

$$2T = \sum^{6} J_{ki}\omega_k\omega_i$$

$$where:$$

$$J_{kk} = \begin{Bmatrix} J_k + m(U_{xk}{}^2 + U_{yk}{}^2 + U_{zk}{}^2) \\ + J_x U_{\psi k}^2 + J_y U_{\theta k}^2 + J_z U_{\phi k}{}^2 \end{Bmatrix} \qquad (3.7)$$

$$k = 1, \ . \ . \ . \ , 6$$

$$J_{ki} = \begin{Bmatrix} m\left(U_{xi}U_{xk} + U_{yi}U_{yk} + U_{zi}U_{zk}\right) \\ + J_x U_{\psi i}U_{\psi k} + J_y U_{\theta i}U_{\theta k} + J_z U_{\phi i}U_{\phi k} \end{Bmatrix}$$

$$k \neq i \ , k = 1,\ldots,5. \ \ , i = 2,\ldots,6.$$

Differentiation of the expression of the kinetic energy (3.7) of six degrees of freedom docking manipulator spacecraft on six input angles of rotation of the input links gives:

$$\frac{\partial T}{\partial \alpha_i} = \frac{1}{2}\sum_{k=1}^{6} \frac{\partial J_{ki}}{\partial \alpha_i}\omega_k\omega_i \ , \ \ i = 1, \ . \ . \ . \ , 6 \qquad (3.8)$$

Note that in the resulting six equations, the inertia coefficients depends on the relations (3.5), which in turn depend on the input angles (3.3). We can write the partial derivative inertial forces (3.7) on the input coordinates of the considered manipulator as follows:

$$\frac{\partial J_{kk}}{\partial \alpha_i} = 2\left[\begin{array}{l} m\left(U_{xk}\dfrac{\partial U_{xk}}{\partial \alpha_i} + U_{yk}\dfrac{\partial U_{yk}}{\partial \alpha_i} + U_{zk}\dfrac{\partial U_{zk}}{\partial \alpha_i}\right) \\ + J_x U_{\psi k}\dfrac{\partial U_{\psi k}}{\partial \alpha_i} + J_y U_{\theta k}\dfrac{\partial U_{\theta k}}{\partial \alpha_i} + J_z U_{\phi k}\dfrac{\partial U_{\phi k}}{\partial \alpha_i} \end{array}\right]$$

$$where$$

$$k = 1,\ldots,6 \quad i = 1,\ldots,6$$

$$\frac{\partial J_{ki}}{\partial_{\alpha i}} = \left\{\begin{array}{l} m\left(\begin{array}{l} U_{xi}\dfrac{\partial U_{xk}}{\partial \alpha_i} + U_{xk}\dfrac{\partial U_{xi}}{\partial \alpha_i} + U_{yi}\dfrac{\partial U_{yk}}{\partial \alpha_i} \\ + U_{yk}\dfrac{\partial U_{yi}}{\partial \alpha_i} + U_{zi}\dfrac{\partial U_{zk}}{\partial \alpha_i} + U_{zk}\dfrac{\partial U_{zi}}{\partial \alpha_i} \end{array}\right) \\ + J_x\left(U_{\psi i}\dfrac{\partial U_{\psi k}}{\partial \alpha_i} + U_{\psi k}\dfrac{\partial U_{\psi i}}{\partial \alpha_i}\right) \\ + J_y\left(U_{\theta i}\dfrac{\partial U_{\theta k}}{\partial \alpha_i} + U_{\theta k}\dfrac{\partial U_{\theta i}}{\partial \alpha_i}\right) \\ + J_z\left(U_{\phi i}\dfrac{\partial U_{\phi k}}{\partial \alpha_i} + U_{\phi k}\dfrac{\partial U_{\phi i}}{\partial \alpha_i}\right) \end{array}\right\} \tag{3.9}$$

$$where$$

$$k \neq 1; k = 1, \ldots, 5; i = 2, \ldots, 6$$

Differentiating the expression of the kinetic energy (3.7) on the angular velocity and having simple mathematical operations, we obtain:

$$\frac{\partial T}{\partial \omega_i} = \sum_{k=1}^{6} J_{ki}\omega_k \quad , i = 1, \ldots, 6 \tag{3.10}$$

Subsequent differentiation of the expression of the kinetic energy (3.10) in time has the form of:

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \omega_i}\right)=\sum_{k=1}^{6} J_{ki}\varepsilon_k+\sum_{k=1}^{6}\frac{\partial J_{ki}}{\partial \alpha_i}\omega_k^2 \quad ;i=1,...,6 \qquad (3.11)$$

Supplying values of the derivatives (3.8) and (3.11) into equations of Lagrange-Euler (3.1), we get:

$$\sum_{k=1}^{6} J_{ki}\varepsilon_k+\sum_{k=1}^{6}\frac{\partial J_{ki}}{\partial \alpha_i}\omega_k^2-\frac{1}{2}\sum_{k=1}^{6}\frac{\partial J_{ki}}{\partial \alpha_i}\omega_{ki}\omega_i=M_{bi}; \quad i=1,...,6 \qquad (3.12)$$

Equation (3.12) is the equation of motion the six degrees of freedom parallel structure docking spatial manipulator with variable inertia coefficients (3.9) depending on the position of the input links.

Using equations (2.4), (2.7) and (3.5), we obtain the expression bringing moment of forces M_{bi} determined from the condition of equality of elementary works on the respective possible movement of all forces applied to the ring of docking manipulator of spacecraft:

$$M_{bi}=\left\{\begin{array}{l} F_x\left(U_{xi}+z_cU_{\theta i}-y_cU_{\phi i}\right) \\ +F_y\left(U_{yi}+x_cU_{\phi i}-z_cU_{\psi i}\right) \\ +F_z\left(U_{zi}+y_cU_{\phi i}-x_cU_{\theta i}\right) \end{array}\right\}; i=1,...,6 \qquad (3.13)$$

Note that each M_{bi} in the equation (3.13) correspond to five independent parameters fixed input links.The equation (3.12) can also be represented in the form:

$$\sum_{k=1}^{6} J_{ki}\ddot{\alpha}_k+\sum_{k=1}^{6}\frac{\partial J_{ki}}{\partial \alpha_i}\dot{\alpha}_k{}^2-\frac{1}{2}\sum_{k=1}^{6}\frac{\partial J_{ki}}{\partial \alpha_i}\dot{\alpha}_k\dot{\alpha}_i=M_{bi} \quad i=1,...,6 \qquad (3.14)$$

The solution of the problem of the dynamics of spatial docking manipulator of spacecraft describes six nonlinear differential equations (3.14) of the second order.

Literature

1. Alizade R.I Modular method of Kinematic analysis and synthesis of spatial linkage mechanisms.Baku, 1992, p.395.
2. R.Alizade .Kinematic analysis of spatial serial and parallel manipulator by using the theory of screws.Technics, Baku, 2000, №1, pp.5-15.
3. V.S.Syromyatnikov .The docking device of spacecraft.Moscow, Mechanical Engineering, 1989, p.216.
4. M.Shahinpoor .Robotics course. Pub.Mir, 1990, p.527.